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Nonlinear Predictive Control of Feedback Linearizable Systems and Flight Control System Design

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The question of output trajectory control of input–output feedback linearizable nonlinear dynamic systems using state variable feedback is considered. For the derivation of the predictive control law, a vector function s is chosen as a linear combination of the tracking error, its higher order derivatives, and the integral of the tracking error. The control law is obtained by the minimization of a quadratic function of the predicted value of s and the control input. It is shown that in the closed-loop system the trajectories are uniformly ultimately bounded in the presence of uncertainty in the system parameters. Based on these results, a flight control system for the trajectory control of the output variables pitch, sideslip, and roll angles (θ, β, ϕ) using aileron, rudder, and elevator control is presented. Simulation results are obtained to show that precise simultaneous longitudinal and lateral maneuvers can be performed in spite of the uncertainty in the aerodynamic parameters.

I. Introduction

THE control of nonlinear systems to follow reference trajectories is an important problem. Input–output and input state feedback linearization techniques based on geometric control theory are useful for the design of control laws for nonlinear systems.[1,2] Using input–output feedback linearization (input–output map inversion), control laws are developed such that in the closed-loop system the tracking error is governed by linear differential equations. However, the feedback linearization technique requires exact knowledge of the system parameters for canceling the system nonlinearities, and the zero dynamics of the system must be stable for the success of the controller. The zero dynamics represent the residual dynamics of the system when the output error is identically zero.[1,2] Also, when control saturation takes place, feedback linearization cannot be accomplished.

In an interesting paper, Lu[3] has considered the design of a predictive controller for nonlinear systems using state variable feedback. For the derivation of the controller, a performance index (PI) that is a quadratic function of the predicted value of the state variables and the control input is optimized. Thus, this approach provides a trade-off between satisfactory tracking performance and the control magnitude requirement. Moreover, the controller can maintain robust performance in the presence of parameter uncertainty. In a recent paper, Khan and Lu[4] have considered a modified performance index that includes a quadratic function of the derivative of the predicted value of the state vector also for the derivation of a predictive controller.

Modern high-performance aircraft operate in a large flight envelope. Maneuvers such as rolling pullouts and high-acceleration turns require a simultaneous, rapid rolling and pitching motion. The dynamic model of aircraft has significant nonlinearity during such maneuvers. In recent years, the design of flight control systems using nonlinear inversion and feedback linearization for performing large maneuvers of aircraft has been reported.[5–12] An approximate tracking controller for aircraft has been designed in Ref. 12.

In this paper, predictive output trajectory control of input–output feedback linearizable systems using state variable feedback is considered. The contribution of this paper lies in the derivation of a new robust predictive controller based on the optimization of a judiciously chosen PI. Unlike Refs. 3 and 4, the selected PI is a quadratic function of the predicted value of a vector function s and the control input. The function s is a linear combination of the tracking error, its higher order derivatives, and its integral. The highest order derivative of the tracking error included in the ith component of vector s is r_i, the relative degree[1,2] of the ith component of the output vector. The advantage of this choice of the PI is that it allows filtering of the tracking error by a stable r_ith-order filter. The design parameters of the filter provide flexibility in shaping the transient responses of the tracking error. Furthermore, the integral feedback term in the filter enhances the steady-state performance. It is shown that the derived controller achieves robust performance in the presence of parameter variation. For the class of systems with unstable zero dynamics, a modification in the PI is made by including in addition a quadratic function of the predicted value of the error in the state vector associated with the zero dynamics for the derivation of the controller. These results are applied to the design of a flight controller for nonlinear roll-coupled maneuvers of aircraft using aileron, rudder and, elevator control. Simulation results for the trajectory control of the output variables pitch, sideslip, and roll angles (θ, β, ϕ) are presented.

This paper is organized as follows. Section II presents the control problem. A predictive control law is derived in Sec. III. Section IV examines the robustness of the controller. The control of aircraft is considered in Secs. V and VI, and Sec. VII presents the numerical results.

II. Problem Formulation

We shall consider a class of systems described by

$$\dot{x} = f(x) + g(x)u \qquad y = c(x) \tag{1}$$

where the state vector $x \in M \subset R^n$, the control input $u \in U \subset R^m$, and the output $y = [y_1, \ldots, y_m]^T \in R^m$. Here M and U are subsets and T denotes transposition. The functions $f(x)$ and $c(x) = [c_1(x), \ldots, c_m(x)]^T$ are assumed to be continuously differentiable a sufficient number of times on M, and $g(x) = (g_1(x), \ldots, g_m(x))$ is a continuous function of x. It is assumed that f, g, and c are bounded on M. (For compactness in notation often the arguments of various functions are suppressed.)

Let a smooth reference trajectory $y_c(t) \in R^m$ be given. We are interested in deriving a control law $u(x, t)$ such that in the closed-loop system $y(t)$ tracks $y_c = [y_{c1}, \ldots, y_{cm}]^T$. That is, the tracking

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error $e(t) = [e_1, \dots, e_m]^T = y(t) - y_c(t)$ tends to zero as $t \to \infty$.

In this paper, it is assumed that the class of systems (1) is input–output feedback linearizable.[1,2] Define the Lie derivatives of $c_i(x)$ and any smooth function $\kappa(x)$ with respect to $f(x)$ and $g(x)$ as

$$
\begin{aligned}
L_f^0 c_i &= c_i \\
L_f c_i &= \frac{\partial c_i}{\partial x} f \\
L_f^j c_i &= L_f\left(L_f^{j-1} c_i\right) \\
L_g \kappa &= \left(L_{g_1}\kappa, \dots, L_{g_m}\kappa\right) \\
L_g c &= \left((L_g c_1)^T, \dots, (L_g c_m)^T\right)^T
\end{aligned}
\tag{2}
$$

Let the relative degree of y_i be r_i. The relative degree r_i is defined to be the least nonnegative integer j such that the jth derivative $y_i^{(j)} = \mathrm{d}^j y_i/\mathrm{d}t^j$ of y_i along the trajectory of Eq. (1) explicitly depends on u for the first time.[1] Then, derivatives of y_i are given by $(i = 1, \dots, m)$

$$
\begin{aligned}
y_i^{(j)} &= L_f^j c_i & j &= 1, \dots, r_i - 1 \\
y_i^{(j)} &= L_f^j c_i + \left(L_g L_f^{j-1} c_i\right) u & j &= r_i
\end{aligned}
\tag{3}
$$

Define the decoupling matrix D as $D = (D_1^T, \dots, D_m^T)^T$, $D_i = L_g L_f^{r_i - 1} c_i$, and let $a(x) = [a_1, \dots, a_m]^T$, $a_i(x) = L_f^j c_i$, $i = 1, \dots, m$, and $Y_r = (y_1^{(r_1)}, \dots, y_m^{(r_m)})^T$. It is assumed that $D(x)$ is invertible at each $x \in M$. Under this assumption, feedback linearization of the input–output map can be accomplished, and each component of the output vector can be independently controlled by inputs.[1,2] Using Eq. (3) gives

$$
Y_r = a(x) + D(x)u \tag{4}
$$

This expression for Y_r will be used in the following section for the derivation of the control law.

III. Predictive Control

In this section, the derivation of a predictive control law for input–output feedback linearizable systems is considered. Define a vector function $s = (s_1, \dots, s_m)^T$, where, for $i = 1, \dots, m$,

$$
\begin{aligned}
s_i &= e_i^{(r_i - 1)} + k_{ir_i - 1} e_i^{(r_i - 2)} + \dots + k_{i1} e_i + k_{i0} x_{si} \\
\dot{x}_{si} &= e_i
\end{aligned}
\tag{5}
$$

where the gains k_{ij} are positive constants chosen such that the polynomial

$$
p_i(\lambda) = \lambda^{r_i} + k_{ir_i - 1}\lambda^{r_i - 1} + \dots + k_{i1}\lambda + k_{i0} = \prod_{j=1}^{r_i} (\lambda - \lambda_{ij}) \tag{6}
$$

is Hurwitz. The function s_i is a linear combination of error e_i, its derivatives, and includes an integral term as well. According to Eq. (5), the tracking error e_i is the output of an r_ith-order stable filter with input s_i. The parameters $k_{ij}(\lambda_{ij})$ are chosen to shape the tracking error responses. Differentiating Eq. (5) gives

$$
\dot{s} = Y_r - Y_c + z \tag{7}
$$

where $z_i = (k_{ir_i - 1} e_i^{(r_i - 1)} + \dots + k_{i1}\dot{e}_i + k_{i0} e_i)$, $z = (z_1, \dots, z_m)^T$, and $Y_c = (y_{c1}^{(r_1)}, \dots, y_{cm}^{(r_m)})^T$.

For the derivation of the predictive controller, consider a performance index of the form

$$
J[u] = \tfrac{1}{2}[s^T(t+h) Q s(t+h) + u^T(t) R u(t)] \tag{8}
$$

where Q and R are positive-definite symmetric matrices, $h > 0$ is a small positive number, and $s(t+h)$ is the predicted value of s at the instant $t + h$. The Taylor series expansion of $s(t+h)$ at t using Eqs. (4), (5), and (7) gives

$$
\begin{aligned}
s(t+h) &\approx s(t) + h\dot{s}(t) \\
&= s(t) + h(a + z - Y_c + Du)
\end{aligned}
\tag{9}
$$

Using Eq. (3), one has

$$
z_i(x, t) = \sum_{j=0}^{r_i - 1} k_{ij}\left[L_f^j(c_i) - y_{ci}^{(j)}\right] \tag{10}
$$

Now the control input is chosen so that the performance criterion is minimized. For the minimization of J, one sets $\partial J/\partial u = 0$ in Eq. (8), which, in view of Eq. (9), gives

$$
Ru + hD^T Q[s + h(a + z - Y_c + Du)] = 0 \tag{11}
$$

Solving Eq. (11) gives the predictive control law

$$
u = -h(R + h^2 D^T Q D)^{-1} D^T Q[s + h(a + z - Y_c)] \tag{12}
$$

To obtain a state variable feedback control law, one substitutes z from Eq. (10) and s_i given by

$$
s_i = \sum_{j=1}^{r_i} k_{ij}\left[L_f^{j-1}(c_i) - y_{ci}^{(j-1)}\right] + k_{i0} x_{si} \qquad k_{ir_i} = 1 \tag{13}
$$

in Eq. (12). By a proper selection of weighting matrices Q and R, the desirable response characteristics of the tracking error are obtained. Larger values of Q can reduce the tracking error magnitude.

One can easily show that the predictive control law (12) tends to a decoupling control law as the matrix R tends to a null matrix. Setting $R = 0$ in Eq. (12), substituting the resulting equation in Eq. (4), and noting that D^{-1} exists, one has

$$
Y_r = -h^{-1}s - (z - Y_c) \tag{14}
$$

In view of Eq. (7), Eq. (14) implies that $(i = 1, \dots, m)$

$$
\dot{s}_i + s_i/h = 0 \tag{15}
$$

We observe from Eqs. (15) and (5) that the responses for each of s_i and e_i are decoupled, and $s(t) = e^{-h^{-1}t}s(0)$ tends to zero as $t \to \infty$ for any $s(0)$. Furthermore, for matched initial conditions given by $(i = 1, \dots, m)$

$$
x_{si}(0) = 0, \qquad e_i^{(j)}(0) = 0 \qquad j = 0, 1, \dots, r_i - 1 \tag{16}
$$

one has $s(0) = 0$. This implies that $s_i(t) \equiv 0$ and $\dot{s}_i(t) = 0$. Thus, in view of Eq. (5), it follows that $e(t) \equiv 0$, which implies perfect tracking of y_c. The controller derived in Ref. 3 has a similar decoupling property.

The eigenvalues associated with the system (5) and (15) are $\lambda_{ij}, h^{-1}, j = 1, \dots, r_i$. Thus only one mode of e_i is a function of h, and the remaining modes are functions of λ_{ij}. For the case when h is sufficiently small, system (5) and (15) is a singularly perturbed[13] system for which $e_i^{(j)}$, $j = 0, 1, \dots, r_i - 2$, and x_{si} are the slow variables and s_i is the fast variable $(i = 1, \dots, m)$. For such a case, the solution $e_i(t)$ of Eq. (5) is well approximated by the solution $\bar{e}_i$ of a simplified system obtained from Eq. (5) by setting $s_i(t) = 0$. That is,

$$
e_i(t) - \bar{e}_i = \mathcal{O}(h)
$$

Thus the modes of e_i are almost solely determined by the chosen parameters λ_{ij} since $e_i \approx \bar{e}_i$.

For input–output linearizable systems, one can define a coordinate transformation[1]

$$
\xi = \left[L_f^0 c_1, \dots, L_f^{r_1 - 1} c_i; \dots; L_f^0 c_m, \dots, L_f^{r_m - 1} c_m; \eta^T\right]^T \in R^n
$$

where $\eta \in R^{n_0}$, $n_0 = n - \sum_{i=1}^m r_i$. The subvector η is said to be associated with the zero dynamics of the system. The zero dynamics are defined as the dynamics of the closed-loop system (1) and (12) when $e(t) \equiv 0$. As usual, for the output trajectory control, it is assumed here that in the closed-loop system the state component η remains bounded for the chosen values of R. However, if the trajectories of the zero dynamics are unstable or unacceptable, a quadratic function of the predicted value of the tracking error in η is included in the PI for the derivation of the control law in order to obtain satisfactory responses. Since for the aircraft model considered here bounded trajectories are obtained, the details of derivation are not presented.

It has been shown that, as $R \to 0$, decoupled responses of e_i are obtained. Since η is assumed to have bounded solutions for the chosen values of R, using an argument related to continuous dependence of solutions of differential equations (Ref. 14) on system parameters, it can be shown that, for nonzero but small values of R, the responses for e_i remain approximately decoupled.

The predictive controller of this paper has been derived under the assumption that control saturation does not occur. As also observed in Refs. 3 and 4, it is interesting to note that when the control magnitude exceeds its limit, the use of maximum (or minimum) allowable control is the best choice within the control bounds that minimize the chosen PI. In contrast, feedback linearization cannot be accomplished whenever the control magnitude exceeds the permissible limit, and one encounters difficulty in modifying the controller designed using nonlinear inversion theory to obtain a satisfactory performance.

IV. Robustness of Control System

Suppose the perturbed system (1) is given by

$$\begin{gathered} \dot{x} = f^*(x) + \Delta f(x) + [g^*(x) + \Delta g(x)]u \\ y = c(x) \end{gathered} \tag{17}$$

where f^* and g^* are the functions computed using the nominal values of the parameters and Δf and Δg denote the uncertain terms. We assume that the relative degree of y_i is r_i for all parametric perturbations under consideration. In such a case, each of the functions in Eq. (11) can be written as

$$\begin{aligned} D &= D^* + \Delta D \qquad & a &= a^* + \Delta a \\ z &= z^* + \Delta z \qquad & s &= s^* + \Delta s \end{aligned} \tag{18}$$

where the starred quantities correspond to the nominal system and uncertain portions are lumped in terms with coefficient Δ.

First, let us assume that $R = 0$. Then the control law (12), computed using the nominal parameters, simplifies to

$$u = -h^{-1}(D^*)^{-1}[s^* + h(a^* + z^* - Y_c)] \tag{19}$$

Substituting Eq. (19) in Eq. (4) gives

$$Y_r = a - h^{-1}D(D^*)^{-1}[s^* + h(a^* + z^* - Y_c)] \tag{20}$$

Substituting Eq. (20) in Eq. (7) and simplifying give

$$\dot{s} = -h^{-1}s - h^{-1}\beta_0 s + \eta_1 + h^{-1}\eta_2 \tag{21}$$

where

$$\eta_1(x,t) = \Delta a + \Delta z - \beta_0(a^* + z^* - Y_c)$$

$$\eta_2(x,t) = (I + \beta_0)\Delta s \qquad \beta_0 = \Delta D(D^*)^{-1}$$

It is assumed that the perturbations are such that

$$\|\beta_0(x)\| \le \gamma_0 < 1 \qquad x \in M \tag{22}$$

By the assumption of boundedness and differentiability of functions f, g, and c on M, it follows that η_1 and η_2 are bounded. Then using a Lyapunov function $W = (s^T s/2)$, it can be shown that

$$\dot{W} \le h^{-1}(1-\gamma_0)\|s\|[\|s\| - \mu(h\|\eta_1\| + \|\eta_2\|)] \tag{23}$$

where $\mu = 1/(1-\gamma_0)$. Thus $\dot{W} < 0$ if $\|s\| > \mu(h\|\eta_1\| + \|\eta_2\|)$. This implies that $s(t)$ is uniformly bounded, and the trajectory evolves such that $s(t)$ is confined in a set of ultimate boundedness B^s of radius r^* for all $t > t^s$, a finite time,[15] where

$$B^s = \{s: \|s\| < r^*, r^* > \mu(h\|\eta_1\| + \|\eta_2\|)\} \tag{24}$$

In view of Eq. (5), the transfer function relating e_i as an output and s_i as an input is given by

$$F_i(\tilde{s}) = \frac{\tilde{s}}{\tilde{s}^{r_i} + k_{ir_i-1}\tilde{s}^{r_i-1} + \cdots + k_{i1}\tilde{s} + k_{i0}} \tag{25}$$

where $\tilde{s}$ is the Laplace variable. Since $F_i(\tilde{s})$ is a strictly stable filter and the input $s_i(t)$ to the filter has been shown to be bounded, it follows that e_i is bounded. The feedback gains k_{ij} provide additional flexibility in shaping the tracking error responses. In view of Eq. (25), it follows that if $s(t)$ tends to a constant function, then $e(\infty) = 0$ due to error integral feedback.

In view of Eq. (24), the set of ultimate boundedness B^s can be made arbitrarily small if $\eta_2 \to 0$ and $h \to 0$. However, this will require high-gain feedback. The function η_2 is zero if perfect measurement of $e_i^{(j)}$, $j = 0, 1, \ldots, r_i - 1$, is available. Furthermore, for $R \neq 0$, using an argument on continuous dependence of solutions on R, one concludes that in the closed-loop system the error e remains bounded.

V. Aircraft Model

For studying the dynamics of the aircraft, its principal axes are chosen as body axes. The equations of motion of the aircraft are given by (see Refs. 16 and 17 for the derivation and the notation)

$$\begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{\phi} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} l_\beta\beta + l_q q + l_r r + (l_{\beta\alpha}\beta + l_{r\alpha}r)\Delta\alpha + l_p p - i_1 qr \\ \bar{m}_\alpha\Delta\alpha + \bar{m}_q q + i_2 pr - m_{\dot{\alpha}} p\beta + m_{\dot{\alpha}}(g/V)(\cos\theta\cos\phi - \cos\theta_0) \\ n_\beta\beta + n_r r + n_p p + n_{p\alpha} p\Delta\alpha - i_3 pq + n_q q \\ q - p\beta + z_\alpha\Delta\alpha + (g/V)(\cos\theta\cos\phi - \cos\theta_0) \\ y_\beta\beta + p(\sin\alpha_0 + \Delta\alpha) - r\cos\alpha_0 + (g/V)\cos\theta\sin\phi \\ p + q\tan\theta\sin\phi + r\tan\theta\cos\phi \\ q\cos\phi - r\sin\phi \end{pmatrix} + \begin{pmatrix} \tilde{l}_{\delta a} & l_{\delta r} & 0 \\ 0 & 0 & \bar{m}_{\delta e} \\ \tilde{n}_{\delta a} & n_{\delta r} & 0 \\ 0 & 0 & z_{\delta e} \\ y_{\delta a} & y_{\delta r} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \delta a \\ \delta r \\ \delta e \end{pmatrix} \triangleq f(x) + g(x)u \tag{26}$$

where $\Delta\alpha = \alpha - \alpha_0$, the state vector $x = [p, q, r, \alpha, \beta, \phi, \theta]^T$, the control vector $u = [\delta a, \delta r, \delta e]^T$, $\tilde{l}_{\delta a} = l_{\delta a} + l_{\alpha\delta a}\Delta\alpha$, and $\tilde{n}_{\delta a} = n_{\delta a} + n_{\alpha\delta a}\Delta\alpha$. Let

$$f(x) = [f_p(x), f_q(x), f_r(x), f_\alpha(x), f_\beta(x), f_\phi(x), f_\theta(x)]^T$$

Here the speed V is assumed constant, and the model contains only a rudimentary representation of the aerodynamic nonlinearities. The assumption of constant speed in large maneuver is unrealistic. However, these simplifications are in no way essential and are used only to make the example more tractable. Speed could be considered as variable and throttle control could be included in the input vector u for controller design, whereas introducing more complete nonlinear dynamics would simply increase the computational difficulties.

Suppose that smooth reference trajectories β_c, ϕ_c, and θ_c for the sideslip, roll, and pitch angles are given. We are interested in deriving a predictive control law such that in the closed-loop system the pitch, sideslip, and roll angles follow the given reference trajectories θ_c, β_c, and ϕ_c.

VI. Flight Control System

In this section, a flight control system is designed for the control of the output vector (θ, β, ϕ). Let the reference output vector be $y_c = (\theta_c, \beta_c, \phi_c)^T$. The output vector chosen for maneuver is

$$y = c(x) = (\theta, \beta, \phi)^T \tag{27}$$

To this end, feedback linearization of the input(u)–output(y) map is examined by the application of the inversion algorithm.[1,2,18] Taking the derivative of y along the solution of Eq. (26) gives

$$\dot{y} = L_f c + (L_g c)u \tag{28}$$

where $L_f c = (f_\theta, f_\beta, f_\phi)^T$ and

$$L_g c = \begin{pmatrix} 0 & 0 & 0 \\ y_{\delta a} & y_{\delta r} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{29}$$

Since aileron, rudder, and elevator are principally moment-producing devices, the elements $y_{\delta a}$ and $y_{\delta r}$ are quite small. For the controller design, we neglect these terms by setting $y_{\delta a} = y_{\delta r} = 0$ in $\dot{y}$. Thus Eq. (28) gives

$$\dot{y} = L_f c \tag{30}$$

Differentiating Eq. (30) gives

$$\ddot{y} = L_f^2 c + (L_g L_f c)u \tag{31}$$

Since $L_g L_f c_i \neq 0$ $(i = 1, 2, 3)$, $r_i = 2$, $a = L_f^2 c$, $D = (D_1^T, D_2^T, D_3^T)^T = L_g L_f c$, where

$$L_f^2 c = [L_f f_\theta, L_f f_\beta, L_f f_\phi]^T$$

$$D_1 = [-\tilde{n}_{\delta a}\sin\phi, -n_{\delta r}\sin\phi, \bar{m}_{\delta e}\cos\phi]$$

$$D_2 = y_\beta[y_{\delta a}, y_{\delta r}, 0] + \sin(\alpha_0 + \Delta\alpha)[\tilde{l}_{\delta a}, l_{\delta r}, 0] \tag{32}$$

$$+ p[0, 0, z_{\delta e}] + \cos\alpha_0[\tilde{n}_{\delta a}, n_{\delta r}, 0]$$

$$D_3 = [\tilde{l}_{\delta a}, l_{\delta r}, 0] + \tan\theta\sin\phi[0, 0, \bar{m}_{\delta e}] + \tan\theta\cos\phi[\tilde{n}_{\delta a}, n_{\delta r}, 0]$$

The matrix D is nonsingular at each $x \in M_1$, a subset of R^7 in which the determinant of D is nonzero. The input–output map of the system is linearizable on M_1. We shall be interested in the trajectory of the system evolving on M_1. The function s is then given by

$$s = \dot{e} + K_1 e + K_0 x_s \qquad \dot{x}_s = e \tag{33}$$

where $e = [(\theta - \theta_c), (\beta - \beta_c), (\phi - \phi_c)]^T$, $K_j = \mathrm{diag}(k_{1j}, k_{2j}, k_{3j})$, $j = 0, 1$. In this case $Y_c = [\ddot{\theta}_c, \ddot{\beta}_c, \ddot{\phi}_c]^T$, and

$$z = K_1(L_f c - \dot{y}_c) + K_0 e \qquad s = (L_f c - \dot{y}_c) + K_1 e + K_0 x_s \tag{34}$$

Substituting a and D from Eq. (32) and z and s from Eq. (34) in Eq. (12) gives the required control.

VII. Simulation Results

In this section, simulation results are presented for the aircraft model studied in Refs. 16 and 17 for the two flight conditions, namely for the flight condition 1 (FC1): $M = 0.9$, $H = 20{,}000$ ft, and for FC2: $M = 0.7$, $H = 0$ ft. The complete set of aerodynamic parameters is provided in Refs. 16 and 17, and $\alpha_0 = 1.5$ deg and $\theta_0 = 0$.

The command generator is assumed to be of the form

$$\Pi(\hat{D})[y_c(t) - y^*(t)] = 0 \tag{35}$$

where $\hat{D} = \mathrm{d}/\mathrm{d}t$; $\Pi(\hat{D}) = (\hat{D} + \lambda_c)^3$, $\lambda_c > 0$, and y^* is the external input. The initial conditions chosen for the command generator are $y_c(0) = \dot{y}_c(0) = \ddot{y}_c(0) = 0$, except $\alpha_c(0) = 1.5$ deg. The selected

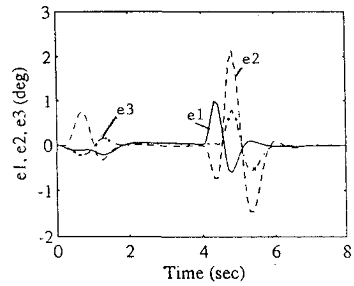

Fig. 1 Control of (θ, β, ϕ) (controller of FC2, $q_0 = 1$), tracking error.

feedback matrices are $K_1 = 10I$, $K_0 = 5I$, $\lambda_c = 3, 3.5$, $h = 0.1$, $R = I$, $Q = q_0 I$, and $q_0 = 1, 100$. Here I denotes an identity matrix of appropriate dimension. These feedback gains were selected by examining the simulated responses after several trials. Let the equilibrium state of the aircraft be $x^* = (0, 0, 0, 1.5\ \mathrm{deg}, 0, 0, 0)^T$. In this section, the peak values of u and e are denoted as u_m and e_m, respectively.

It is desired to control the pitch and the roll angles from the initial value (0, 0) to (40, 80) (deg) and then to (0, 0). Furthermore, the sideslip angle has to be regulated close to zero. For this purpose, we set y^* to $y^*(t) = (40, 0, 80)^T$ (deg) for $t \in [0, 4]$ s and $y^* = 0$ for $t > 4$ s in Eq. (35) to generate the reference trajectory.

A1.1 (θ, β, ϕ) control, controller of FC2, small q_0. For trajectory tracking of (θ, β, ϕ), the controller was designed using the parameters of the aircraft at FC2, and the complete closed-loop system (26) and (12) was simulated. The plots of the tracking error responses for the aircraft at FC2 with the nominal initial condition $x(0) = x^*$, $\lambda_c = 3.5$, and $q_0 = 1$ are shown in Fig. 1. The pitch and roll angles smoothly attain (40, 80) (deg) and then return to zero. The state vector x asymptotically converges to x^*, and the trajectory error converges to zero. The response time is of the order of 2 s. The maximum values are $e_m = (0.984, 2.14, 0.802)^T$(deg) and $u_m = (10.70, 20.19, 12.24)^T$ (deg). We observe only a small tracking error in the pitch and the roll angles. However, the error in the sideslip angle is relatively large (2.14 deg). The reason for a larger tracking error in the sideslip angle is that the control law has been derived by neglecting the small contribution of the control forces in the $\dot{\beta}$ equation, but these control forces have been reintroduced in the model of the aircraft for obtaining realistic simulation results.

A1.2 (θ, β, ϕ) control, $q_0 = 100$. To examine the effect of q_0, its value was increased to 100 for simulation. Selected responses are shown in Figs. 2a–f. Since the weighting matrix Q associated with tracking error is considerably larger compared to case A1.1, a significant reduction in the peak values of the tracking error responses are observed. The maximum tracking errors and control magnitudes now are $e_m = (0.016, 0.048, 0.007)^T$ (deg) (Fig. 2e) and $u_m = (9.47, 20.97, 17.49)^T$(deg) (Fig. 2c), respectively. The sideslip angle error is less than 0.05 (deg) (Fig. 2d). The pitch and the roll angles smoothly attain 40 and 80 deg and then converge to zero (Fig. 2b). The responses for the angular velocities converge to zero at the completion of the maneuver (Fig. 2a), and the angle of attack also returns to the equilibrium value (1.5 deg) (Fig. 2f).

A1.3 (θ, β, ϕ) control, off-nominal FC1. The sensitivity of the controller was examined by simulating the aircraft model at FC1 using the controller designed at FC2 of case A1.2. In spite of the perturbed parameters of the aircraft, smooth trajectory tracking was accomplished (Fig. 3), and zero steady-state error was obtained. However, larger control and transient error were observed. The maximum values are $e_m = (1.229, 0.29, 1.54)^T$ (deg) and $u_m = (12.29, 30.56, 25.66)^T$(deg).

To examine the effect of slower commands, simulation was done as in case A1.3 but with smaller value of $\lambda_c = 3$ instead of $\lambda_c = 3.5$. As expected, smaller values $e_m = (1.08, 0.22, 1.51)^T$ (deg) and $u_m = (9.45, 22.49, 23.84)^T$ (deg) were obtained. These results are not shown here in order to save space.

A1.4 (θ, β, ϕ) control, nonzero $e(0)$. To examine the effect of initial tracking error, simulation was done as in case A1.2 except $(\theta(0), \beta(0), \phi(0)) = (3, 0.5, 2)$ (deg). This resulted in an

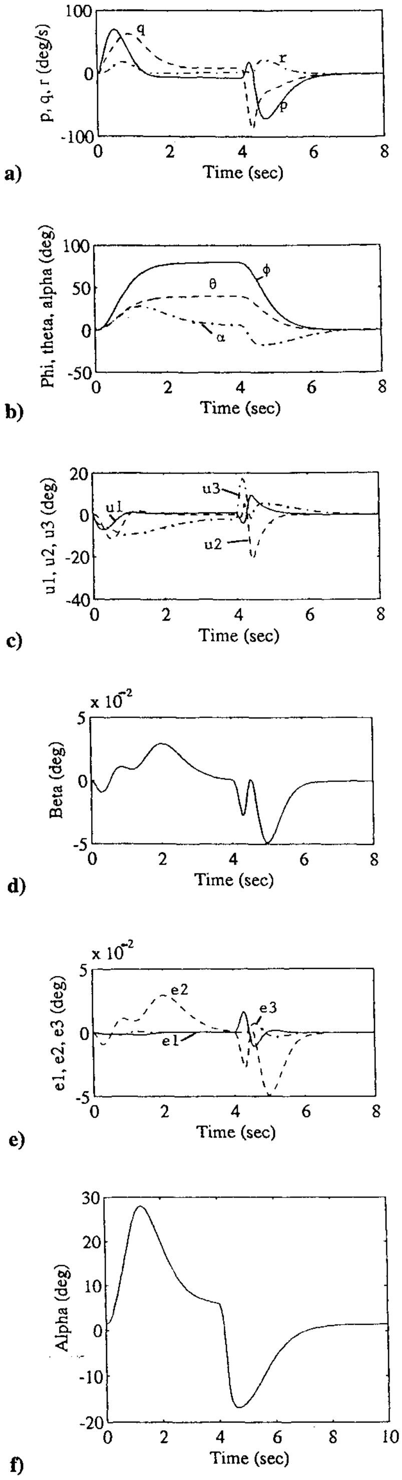

Fig. 2 Control of (θ, β, ϕ) (controller of FC2, q_0 = 100): a) p, q, r; b) ϕ, θ, α; c) u_1, u_2, u_3; d) sideslip angle; e) e_1, e_2, e_3; and f) angle of attack.

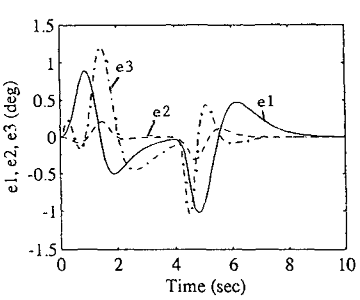

Fig. 3 Control of (θ, β, ϕ) (controller of FC2, q_0 = 100), off-nominal FC1, tracking error.

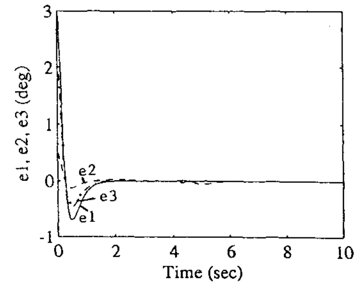

Fig. 4 Control of (θ, β, ϕ) (controller of FC2, q_0 = 100), nonzero e(0), tracking error.

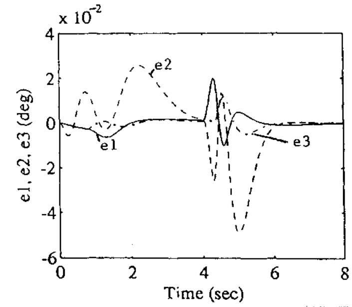

Fig. 5 Control of (θ, β, ϕ) (controller of FC1, q_0 = 100), tracking error.

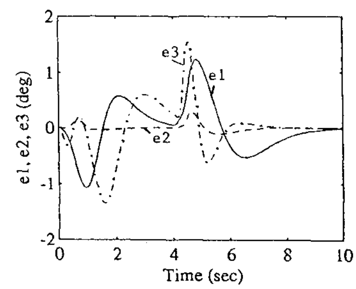

Fig. 6 Control of (θ, β, ϕ) (controller of FC1, q_0 = 100), off-nominal FC2, tracking error.

initial tracking error of $e(0) = (3, 0.5, 2)^T$ (deg). In spite of the initial tracking error, smooth trajectory control was observed (Fig. 4). The maximum values are $e_m = (3, 0.5, 2)^T$ (deg) and $u_m = (9.49, 20.93, 17.57)^T$ (deg).

A2.1 (θ, β, ϕ) control, controller of FC1. Simulation was done with the controller designed using parameters of aircraft at FC1. The feedback parameters of case A1.2 were retained. Simulated tracking error responses for the aircraft at FC1 with $x(0) = x^*$, $\lambda_c = 3.5$, and $q_0 = 100$ are shown in Fig. 5. Smooth tracking is observed. The maximum values are $e_m = (0.02, 0.04, 0.01)^T$ (deg) and $u_m = (11.21, 28.91, 25.55)^T$ (deg). The control magnitude required for maneuver in this case is larger compared to case A1.2. The reason for this is that the control derivatives have smaller magnitude at this flight condition (see Refs. 16 and 17 for the tabulated values of the aerodynamic parameters), and therefore, larger control surface deflections are required to produce the desired moments.

A2.2 (θ, β, ϕ) control, off-nominal FC2. Simulation was done using the aircraft model at the off-nominal FC2 and the controller of case A2.1 designed for FC1. Again accurate trajectory control was accomplished (Fig. 6). The maximum values are $e_m = (1.02, 0.305, 1.21)^T$ (deg) and $u_m = (8.7, 19.32, 17.6)^T$ (deg). Comparing the maximum tracking errors in the off-nominal cases A1.3 and A2.2, it is observed that these errors are of the same order. This suggests that one can design the controller using the parameters of either flight condition for implementation.

Remark 1. Following the derivation procedure of Sec. VI, a predictive control law for the control of the output vector (α, β, ϕ) was also obtained, and simulation results for large maneuvers were obtained. It was observed that precise, robust trajectory control of angle of attack, sideslip angle, and roll angle is accomplished in the closed-loop system. (These results are not shown here in order to save space.)

VIII. Conclusion

In this paper predictive control of nonlinear input–output feedback linearizable systems using state variable feedback was considered. A control law was derived by minimizing a PI that is a quadratic function of the predicted value of a chosen vector function s and the control input. The tracking error is the output of a stable filter. The parameters of this filter can be chosen to shape the tracking error responses. Some results related to boundedness of the tracking error were established. These results were applied to the design of control systems for the trajectory control of (θ, β, ϕ) using aileron, rudder, and elevator control surfaces. Extensive simulations showed that nonlinear precise roll-coupled maneuvers of aircraft can be accomplished in the closed-loop system in spite of the uncertainty in the aerodynamic parameters.

References

[1]Isidori, A., *Nonlinear Control Systems*, Springer–Verlag, Berlin, 1985.

[2]Nijmeijer, H., and Van Der Schaft, A. J., *Nonlinear Dynamical Control Systems*, Springer–Verlag, New York, 1990.

[3]Lu, P., "Nonlinear Predictive Controllers for Continuous Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 3, 1994, pp. 553–560.

[4]Khan, M. A., and Lu, P., "New Technique for Nonlinear Control of Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 5, 1994, pp. 553–560.

[5]Meyer, G., Su, R., and Hunt, L. R., "Application of Nonlinear Transformations to Automatic Flight Control," *Automatica*, Vol. 20, No.1, 1984, pp. 103–107.

[6]Romano, J. J., and Singh, S. N., "I-O Map Inversion, Zero Dynamics and Flight Control," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 26, No. 6, 1990, pp. 1022–1029.

[7]Menon, P. K. A., Badgett, M. E., Walker, R. A., and Duke, E. L., "Nonlinear Flight Test Trajectory Controllers for Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 10, No. 1, 1987, pp. 67–72.

[8]Lane, S. H., and Stengel, R. F., "Flight Control Design Using Nonlinear Inverse Dynamics," *Automatica*, Vol. 24, No. 4, 1988, pp. 471–484.

[9]Snell, S. A., Enns, D. F., and Garrard, W. L., "Nonlinear Inversion Flight Control for a Supermaneuverable Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 4, 1992, pp. 976–984.

[10]Azam, M., and Singh, S. N., "Invertibility and Trajectory Control for Nonlinear Maneuvers of Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 1, 1994, pp. 192–200.

[11]Elgersma, M. R., "Control of Nonlinear Systems and Application to Aircraft," Ph.D. Dissertation, Dept. of Electrical Engineering, Univ. of Minnesota, Minneapolis, MN, 1988.

[12]Hauser, J. E., Sastry, S., and Meyer, G., "Nonlinear Control Design for Slightly Nonminimum Phase System: Application to V/STOL Aircraft," *Automatica*, Vol. 28, No. 4, 1922, pp. 665–680.

[13]Kokotovic, P. V., Khalil, H. K., and O'Reilly, J., *Singular Perturbations Methods in Control: Analysis and Design*, Academic, New York, 1986.

[14]Hale, J. K., *Ordinary Differential Equations*, Wiley-Interscience, New York, 1964.

[15]Hahn, W., *Stability of Motion*, Springer–Verlag, New York, 1967.

[16]Hacker, T., and Oprisiu, C., "A Discussion of the Roll-Coupled Problem," *Progress in Aerospace Science*, Vol. 15, Pergamon, Oxford, England, 1974, pp. 151–180.

[17]Rhoads, D. W., and Schuler, T. M., "A Theoretical and Experimental Study of Airplane Dynamics in Large-Disturbance Maneuvers," *Journal of Astronautical Sciences*, Vol. 24, July 1957, pp. 507–532.

[18]Slotine, J-J. E., and Li, W., *Applied Nonlinear Control*, Prentice–Hall, Englewood, Cliffs, NJ, 1991.